ON THE MARKOV CHAIN PREDICTION OF ACADEMIC MANPOWER SYSTEM IN PRIVATE UNIVERSITY IN NIGERIA

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Abstract

In this study, we considered a Markovian approach in studying the behavior of the academic staff grade levels transitions in Private University in Nigeria. This is to determine the proportion of staff recruited, promoted and withdrawn from various grade levels in the Private Universities over the years and also to forecast the expected manpower structure of the Universities from 2021/2022 to 2030/2031 Academic Sessions. Secondary data were obtained from the Human Resources Department of a Private University in Nigeria for 2016/2017 to 2020/2021 Sessions and were used to evaluate the method with the help of several existing Markov chain laws, theorems, and formulas. The findings from the analysis show that the Academic Staff Grade transition flow is stationary over the observed time period. Also, it was predicted that, at the beginning of 2030/2031 session, the expected staff structure of the Universities will consist of 3 Graduate Assistants; 25 Assistant Lecturers; 37 Lecturer II; 55 Lecturer I; 36 Senior Lecturers; 7 Readers and 42 Professors if the current recruitment and promotion policies in the University remain unchanged. In conclusion, it was observed that the institution is in need of more Lecturer I, Senior Lecturer and Professors than the number already recruited. While it was also observed that the number of Graduate Assistant, Assistant Lecturer, Lecturer II and Readers were to be reduced to make the system effective.

Keywords: Absorbing chain, Chapman-Kolmogorov Equations, Markov Processes, Random walk, Recurrent state, Time Homogeneity

Introduction

Manpower development is the process by which a country develops and expands its human resource capacities by introducing relevant and technical knowledge, skills, and effectiveness in order to successfully achieve established goals. Managing huge organizations such as industrial cooperatives, governmental administrations, or academic systems requires well-organized manpower development. All of these systems have a diverse workforce with distinct positions and responsibilities. In any organization, the abilities required to complete given duties through special training or extensive work experience are highly valued, however in most developing countries/nations, the shortage of people with vital skills and knowledge for efficient national development is a severe concern. To avoid such problems, future personnel demands must be foreseen well in advance, and corresponding techniques to accomplish the required structure on system behavior are critical for forecasting the future development of manpower structure in complex organizations.

However, such predictions are frequently based on previous experience in large systems, and without adequate mathematical or statistical models and computational tools, knowledge obtained from such experiences is frequently impossible to implement. The rate of transitions between different parts of the system, or the transition probability, is the most basic information that may be utilized to model manpower dynamics. Promotions, transfers between assignments, or wastage and input into the system are all common causes of transitions. In many cases, the models used to predict the future structure of a dynamics system are based on Markov chains and their derivatives such as Semi-Markov chains, both of which are based on the assumption that the rules governing the system cannot be arbitrarily changed.

Additionally, we must be cautious because even minor changes in policies can have significant consequences on the future development of manpower structure. The outcome of a Markov chain model depends on the probabilities within the transition matrix, which is time independent (stationary) or time dependent (nonstationary), and a stationary assumption allows the straight forward calculations of the long run equilibrium distribution of the amounts in different states, changes in transience. This was about the upkeep of a graded personnel structure made up of sub-units of departments. Furthermore, given that these departments have similar grade structures and that the grades are the actual promotion cadres of system units, the factor of interest on which conditions for maintaining ability are based in this system is unit transfer across departments.

Moreover, the goal of this study is to employ Markov analysis, which has been applied in a variety of social settings, to explore the behavior of academic staff grade transitions at a private university in Nigeria. Meehan and Ahmed (1990) proposed a demand model for projecting Human Resource Requirements, while Ezekiel (1991) presented a Markov chain approach in modeling the structure and career prospects of academic staff at university colleges. Skulj et al. (2008) established the usage of Markov chains in modeling the Slovenian Armed Forces workforce, while McClean (1991) observed manpower planning models and their estimation, and Khnoog (1996) examined an Integrated System framework and methodology for manpower planning. Also, Ossai and Uche (2009) discussed the maintainability of departmentalized manpower structures in Markov chain model and William (2009) discussed the probabilities and simulations in Markov chain.

Furthermore, Obogbo *et al.* (2013) predicted the academic manpower system of a Polytechnic Institution in Nigeria, and Agboola (2021) introduced direct equation solving algorithms compositions of lower-upper triangular matrix and Grassmann–Taksar–Heyman for the stationary distribution of Markov chains, while Agboola and Ayoade (2021) analysed the matrix geometric and analytical block numerical iterative methods for stationary distribution in the structured Markov chains. Agboola and Ayinde (2021) investigated the concept of the classification of groups of states, between states that are recurrent and states that are transient, in order to provide some insight into the performance measure analysis for the mean first passage time, Rij, the mean recurrence time of state Rjj as well as recurrence iterative matrix R (k+1) while Agboola and Badmus (2021a) analysed the distribution function of the renewal process and sequence $\{Xn, n \geq i\}$ using the concept of discrete time Markov chain to obtain performance measures and with some properties of the Erlang k, exponential and geometric distributions and concluded that it is not possible for an infinite number of renewals to occur in a finite period of time. and, Agboola and Badmus (2021b) established the application of Runge-kutta and backward differentiation methods for solving transient distribution in Markov chain.

In addition, Agboola (2022a) discussed the decomposition and aggregation algorithmic numerical iterative solution methods for the stationary distribution of Markov chain, while Agboola (2022b) analysed and Applied an Irreducible Periodic Markov chain in solving random walk and gambler's ruin. Agboola and Ayinde (2022) demonstrated the use of successive over-relaxation algorithmic and block numerical iterative solution methods to compute the stationary distributions in Markov chain with the help of some existing Markov chain laws, theorems, and formulas and the normalization principle to obtained the stationary distribution vector's, while Agboola, and Nehad (2022) worked on the application of matrix scaling and powering methods of small state spaces for solving transient distribution in Markov chain.

Agboola *et al*. (2022) computed solutions and algorithms for large state spaces using uniformization methods with the help of matrices operations, Markov chain laws, theorems, and formulas, while Agboola *et al*. (2023) formulated the solutions of stationary distribution in Markov chain using Jacobi iterative method and Gauss-Seidel iterative method with the aid of several already-existing laws, theorems, and formulas of Markov chain and the application of normalization principle and matrix operations. However, in this study, our interest is to determine the proportion of staff recruited, promoted and withdrawn from the various grade levels by employing transitions probabilities, absorbing chain, steady-state probabilities and the limiting distribution of Markov analysis.

Nomenclature

k_i, is the number of staff in cadre i at the beginning of t^{th} session; $k(t) = \sum_{i=1}^{7} k_i$, the total size of staff at the beginning of the t^{th} session; $k_{ij}(t)$, the number of persons who moved from grade level i to grade level j at t^{th} session; k_{i0}, the wastage flow from the i^{th} cadre within the t^{th} session; $k_{oj}(t)$, the recruitment flow to grade level j at the beginning of the t^{th} session; $n_{ij}(t)$, the number of persons who moved from grade i to grade j at t^{th} session; $n_i(t)$, the number of staff in cadre i at the beginning of t^{th} session; p_s, the time homogeneous probability matrix of the Markov chain; p_{ij}, the transition probability of a person in grade level i moving to grade level j within the t^{th} sessions $i, j = 1,2,3,4,5,6,7$; $\left(n_{ij}^k\right)$, $k = 1,2,3,4,5,6,7$ denote the Matrices of the transition counts for each of the cadres ranging from Graduate Assistant to a Professor and $\left(p_{ij}^k\right)$, $k = 1,2,3,4,5,6,7$ represent the estimates of their transition probabilities respectively.

Material and Methods

The study area consisted of analysis of Markov chain prediction of academic manpower system of private University in Nigeria We started with the model description, model assumptions, method of data collection, procedure for estimation of transition probability and determination of the transition probabilities and steady state.

Model Description

The application of probability theory to practical situations involving random phenomena necessitates the study of any discipline of probability theory. In practice, the conceivable outcomes of some random trials are known, but the actual results cannot be anticipated with certainty. Nonetheless, the outcomes follow a statistical regularity condition in which the relative regularities of occurrence of alternative outcomes are relatively predictable. The theory of stochastic processes, for example, is the study of systems that change or evolve in time or space according to probabilistic laws. Stochastic process in a family of random variable t which belong to some index set T. if $Ti = \{t: a < t < b\}$, then, the index set T is continuous parameter stochastic process $X(t)$. But if $T = \{t: 0, +1, + \cdots.\}$, then, T is called the discrete parameter stochastic process $X(t)$. Different forms of stochastic processes exist. The Markov process, for example, is a discrete or continuous set with the property that the probability relations for some future time with respect to a particular present time are solely determined by the history that led to the present state.

For this work, the absorbing chain, limiting distribution of Markov chains are going to be used. Let $i = 1,2, \dots .7$; represent the cadre ranging from Graduate assistant to Professor. Where $t = 1, \dots ,5$ represent the academic session of the university; $t = 1$ stands for 2016/2017 session, $t = 2$ stands for 2017/2018 session, $t = 3$ stands for 2018/2019 session, $t = 4$ stands for 2019/2020 session, $t = 5$ stands for 2020/2021 academic session.

Model Assumption

The following assumptions are made about the recruitment and promotion flow and the transition probability matrices (TPM) denoted by $\boldsymbol{P} = \left[\boldsymbol{P_{ij}}\right]_{\boldsymbol{m \times m}}$**;** where m denotes the units

(a) At the start of the session, recruitment can be made into any of the grades where k_{01} represents the recruitment flow and p_{0j} represents the chance of recruitment such that$\sum_{j=1}^{7} p_{0j} = 1$.

(b) Promotion in the university depends on such factors as the qualification experience and productivity of staff but due to individual differences, The promotion flow k_{ij} is a random variable with independent transition probability p_{ij} or which when summed across the j^{th} rows will produce $\sum_{j=1}^{7} p_{ij} + w_i = 1$

(c) The assumption of an orderly and stable flow indicates that both the initial transition probability $[P_i]$ and the total transition probability matrix (TPM) are stationary over time, implying that the probability matrix is time independent**.**

Procedure for Estimation of Transition Probability

A simple method of estimating the transition probability matrix will be used in this work.

A finite Markov chain of j states $j = (1,2, \dots r)$ are observed until is transitions have taken place. If we let k_{ij} be the frequency of transition from $i\ to\ j$ $(i, j = 1,2, \dots , r)\ and\ \{k_{ij} = k_i\}$. These frequencies can be represented as in table below

Table 1: Frequency of transition from state i to j

	1	2	3	$\cdots$	$r-1$	r	Total
1	k_{11}	k_{12}	k_{13}	$\cdots$	$k_{1(r-i)}$	k_{1r}	k_1
2	k_{21}	k_{22}	k_{23}	$\cdots$	$k_{2(r-i)}$	k_{2r}	k_2
$\ddots$							$\ddots$
R	k_{r1}	k_{r2}	k_{r3}	$\cdots$	$k_{r(r-i)}$	k_{rr}	k_r

The total number of trials can be seen to be n which is in

$$k = \sum_{j=1}^{r} k = \sum_{i=1}^{r}\sum_{j=1}^{r} k_{ij} \qquad (1)$$

The above table is a matrix with the usual notation n_{ij}, where n_{ij} is the element on the i^{th} row and j^{th} column and each row table is $n_i, (i = 1, 2, \cdots, r)$.

The first-row total is the n_0 while the last one is n_r. If we let the time homogeneous probability matrix of the Markov chain be p_s, we are interested in the estimation of the elements of the matrix. This shall be denoted by $p_{ij}(i, j = 1, 2, \cdots, r)$ (2)

Determination of the Transition Probabilities

Using the principle of maximum likelihood to exploit the multinomial distribution of k_{ij} given $k_i(t)$ for each period with probabilities, the statistical technique for Markov chains offers the estimate of p_{ij} as

$$p_{ij} = \frac{k_{ij}}{k_i} \quad i = 1, 2, 3, 4, 5, 6, 7 \qquad (3)$$

If Stationary holds, the pooled estimate becomes

$$P_{ij} = \frac{\sum_{t=1}^{5} n_{ij}(t)}{\sum_{t=1}^{5} n_i(t)} \quad i = 1, 2, 3, 4, 5, 6, 7 \qquad (4)$$

The Skulj et al. (2008) model allows for wastage w, which accounts for those who leave the system, and assumes that the transition matrix P need not be stochastic but only sub-stochastic, which means that the total of rows of the transition matrix may be less than 1, and the difference is wastage W.

$$W_i = 1 - \sum_{i=1}^{m} P_{ij} \qquad (5)$$

As a result, dividing each element in row by the transition probability matrix of the Markov chain p yields

$$P = \begin{pmatrix} P_{11} & P_{12} & P_{13}\cdots & P_{1r} \\ \vdots & & \ddots & \vdots \\ P_{r1} & P_{r2} & P_{r3}\cdots & P_{rr} \end{pmatrix} \qquad (6)$$

Steady-State Probabilities

After a lengthy period of time, the state probabilities of a Markov chain should stabilize. If p is the transition matrix for a k -state ergodic Markov chain, we can obtain the following limiting probability distribution for the chain's states:

$$\lim_{k\to\infty} P^k = \begin{bmatrix} \pi_1 & \pi_2 & \cdots & \pi_n \\ \pi_1 & \pi_2 & \cdots & \pi_n \\ \cdots & \cdots & \cdots & \cdots \\ \pi_1 & \pi_2 & \cdots & \pi_n \end{bmatrix} \qquad (7)$$

As a result, over time, the chance of moving from any state of the chain to state j stabilizes at π_j. As a result, we may write

$$\lim_{k\to\infty} p_{ij}(k) = \pi_j \qquad (8)$$

Where $p_{ij}(k)$ is the ij^{th} element of p^k

Method of Data Collection

This data is obtained from the office of the registrar, human resources unit of a Private University in Nigeria. It had shown the number of academic personnel between the year 2016/ 2017 academic session to the year 2020/2021 academic session in the respective colleges.

Table 2: Number of Academic Personnel for Each College in the 2016/2017 Academic Session

CADRE	COLNAS	COLMAS	COLENG	COLENVS	CENPD	TOTAL
G A	2	1	0	4	1	8
A L	9	10	2	6	2	29
LECTURER II	11	8	8	4	3	34
LECTURER I	13	6	13	7	1	40
SENIOR LEC.	11	8	10	0	0	4
READER	1	0	2	1	0	4
PROFESSOR	5	0	3	1	1	10

Table 3: Number of Academic Personnel for Each College in the 2017/2018 Academic Session

CADRE	COLNAS	COLMAS	COLENG	COLENVS	CENPD	TOTAL
G A	4	4	1	3	0	12
A L	19	6	2	8	2	37
LECTURER II	15	5	5	9	1	35
LECTURER I	14	4	10	3	7	48
SENIOR LEC.	9	2	7	1	0	19
READER	0	0	2	0	1	3
PROFESSOR	4	0	5	0	0	9

Table 4: Number of Academic Personnel for Each College in the 2018/2019 Academic Session

CADRE	COLNAS	COLMAS	COLENG	COLENVS	CENPD	TOTAL
G A	6	9	4	5	2	26
A L	18	9	4	11	4	46
LECTURER II	20	4	7	9	1	41
LECTURER I	9	5	8	4	5	31
SENIOR LEC.	7	1	4	3	0	15
READER	1	0	1	0	1	4
PROFESSOR	4	0	3	1	0	8

Table 5: Number of Academic Personnel for Each College in the 2019/2020 Academic Session

CADRE	COLNAS	COLMAS	COLENG	COLENVS	CENPD	TOTAL
G A	4	7	5	4	2	22
A L	21	9	6	8	3	47
LECTURER II	18	6	5	12	3	44
LECTURER I	12	7	4	7	6	36
SENIOR LEC.	10	3	5	5	1	24
READER	1	0	0	0	2	3
PROFESSOR	4	1	5	3	0	13

Table 6: Number of Academic Personnel for Each College in the 2020/2021 Academic Session

CADRE	COLNAS	COLMAS	COLENG	COLENVS	CENPD	TOTAL
G A	3	6	4	4	2	19
A L	21	12	7	8	4	52
LECTURER II	18	6	8	14	3	49
LECTURER I	14	9	4	4	4	35
SENIOR LEC.	10	3	4	8	1	26
READER	0	0	0	0	2	2
PROFESSOR	5	1	6	2	1	15

Table 7: Number of Academic Personnel for Each Cadre in Each Session

Year	GA	AL	L 2	L 1	SL	RE	PRO
2016/2017	8	29	34	40	22	4	10
2017/2018	12	37	35	48	19	3	9
2018/2019	26	46	41	31	15	4	8
2019/2020	22	47	44	36	24	3	13
2020/2021	19	52	49	35	26	12	15

Table 8: Number of Possible Transition of Academic Personnel in Each Session

	LI				SL			
SESSION	R(LI)	LI	P(LII)	W(LI)	R(SL)	SL	P(LI)	W(SL)
2016/2017	0	40	3	2	2	22	3	2
2017/2018	8	48	8	2	0	19	2	5
2018/2019	1	31	3	22	1	15	4	6
2019/2020	8	36	5	5	8	24	2	1
2020/2021	0	35	6	1	2	26	0	0

	READER				PROFESSOR			
SESSION	R(RE)	RE	P(SL)	W(R)	R(PRO)	PRO	P(PRO)	W(PRO)
2016/2017	0	4	1	1	4	10	1	2
2017/2018	0	3	2	2	2	9	2	2
2018/2019	0	4	1	0	1	8	1	3
2019/2020	0	3	2	2	4	13	1	0
2020/2021	0	12	0	1	2	15	1	0

Key; GA=Graduate Assistant, AL=Assistant lecturer, LII= Lecturer II, LI= Lecturer I, SL=Senior Lecturer, RE=Reader, PRO= Professor, R=Recruitment, P=Promotion and W=Wastages (Retirement, Resignation, Sack or Termination, Death and ill health).

RESULT AND ANALYSIS

Let $P_{ij}(t)$ represents the transition probability of a person in grade i moving to grade j within the t^{th} sessions. $i, j = 1, 2, 3, 4, 5, 6, 7$.

Graduate assistant: $n_{ij}^1 = 68, 10$

Assistant lecturer: $n_{ij}^2 = 159, 17$

Lecturer II: $n_{ij}^3 = 154, 19$

Lecturer I: $n_{ij}^4 = 155, 11$

Senior Lecturer: $n_{ij}^5 = 80, 6$

Reader: $n_{ij}^6 = 14, 5$

Professor: $n_{ij}^7 = 40$

The estimate of the probabilities follows for each cadre

$$P_{ij} = \frac{\sum_{t=1}^{5} n_{ij}(t)}{\sum_{t=1}^{5} n_{i}(t)} \qquad i = 1, 2, 3, 4, 5, 6, 7 \tag{9}$$

Graduate assistant: $P_{ij}^{1} = 68/78, 10/78 \; = \; (0.8718, 0.1282)$

Assistant lecturer: $P_{ij}^{2} \; = 159/176, 17/176 \; = \; (0.9034, 0.0966)$

Lecturer II: $P_{ij}^{3} \; = 154/173, 19/173 = \; (0.8902, 0.1098)$

Lecturer I: $P_{ij}^{4} \; = 155/166, 11/166 = \; (0.9337, 0.0663)$

Senior lecturer: $P_{ij\mathrm{j}}^{5} \; = 80/86, 6/86 = \; (0.9302, 0.0698)$

Reader: $P_{ij}^{6} \; = 14/19, 5/19 \; = \; (0.7368, 0.2632)$

Professor: $P_{ij}^{7} \; = 40/40 \; = \; (1.000)$

Using the collected data as presented in the table 1.0 above. The transition probabilities of the academic staff of the University was summarized for the seven grades level below

$$p_{ij} = \begin{array}{cc} & \begin{array}{ccccccc} GA & AL & LII & LI & SL & RE & PRF \end{array} \\ \begin{array}{c} GA \\ AL \\ LII \\ LI \\ SL \\ RE \\ PRF \end{array} & \begin{pmatrix} 0.8717 & 0.1282 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0,0000 \\ 0.0000 & 0.9034 & 0.0966 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.8902 & 0.1098 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.9337 & 0.0663 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.9302 & 0.0698 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.7368 & 0.2632 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{pmatrix} \end{array}$$

Table 2.0: Transition probabilities matrix (TPM) for the seven academic grades.

The limiting value probability for the above transition probabilities are generated below.

$$p_{ij}^{2} = \begin{array}{cc} & \begin{array}{ccccccc} GA & AL & LII & LI & SL & RE & PRF \end{array} \\ \begin{array}{c} GA \\ AL \\ LI \\ LII \\ SL \\ RE \\ PRF \end{array} & \begin{pmatrix} 0.7599 & 0.2276 & 0.0124 & 0.0000 & 0.0000 & 0.0000 & 0,0000 \\ 0.0000 & 0.8161 & 0.1733 & 0.0106 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.7925 & 0.2003 & 0.0073 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.8718 & 0.1236 & 0.0046 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.8653 & 0.1164 & 0.0184 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.5429 & 0.4571 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{pmatrix} \end{array}$$

$$p_{ij}^{3} = \begin{array}{cc} & \begin{array}{ccccccc} GA & AL & LII & LI & SL & RE & PRF \end{array} \\ \begin{array}{c} GA \\ AL \\ LI \\ LII \\ SL \\ RE \\ PRF \end{array} & \begin{pmatrix} 0.6624 & 0.3030 & 0.0330 & 0.0014 & 0.0000 & 0.0000 & 0,0000 \\ 0.0000 & 0.7373 & 0.2331 & 0.0289 & 0.0007 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.7054 & 0.2740 & 0.0200 & 0.0005 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.8140 & 0.1728 & 0.0120 & 0.0012 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.8049 & 0.1461 & 0.0490 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.4000 & 0.6000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{pmatrix} \end{array}$$

$$p_{ij}^{4} = \begin{array}{cc} & \begin{array}{ccccccc} GA & AS & LI & LII & SL & RE & PRF \end{array} \\ \begin{array}{c} GA \\ AL \\ LII \\ LI \\ SL \\ RE \\ PRF \end{array} & \begin{pmatrix} 0.5774 & 0.3586 & 0.0587 & 0.0049 & 0.0001 & 0.0000 & 0,0000 \\ 0.0000 & 0.6661 & 0.2781 & 0.0526 & 0.0026 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.6280 & 0.3333 & 0.0368 & 0.0018 & 0.0001 \\ 0.0000 & 0.0000 & 0.0000 & 0.7600 & 0.2174 & 0.0209 & 0.0044 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.7487 & 0.1638 & 0.7053 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.2947 & 0.7053 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{pmatrix} \end{array}$$

$$p_{ij}^{5} = \begin{array}{cc} & \begin{array}{ccccccc} GA & AL & LII & LI & SL & RE & PRF \end{array} \\ \begin{array}{c} GA \\ AS \\ LI \\ LII \\ SL \\ RE \\ PRF \end{array} & \begin{pmatrix} 0.5033 & 0.3980 & 0.0869 & 0.0110 & 0.0004 & 0.0000 & 0,0000 \\ 0.0000 & 0.6017 & 0.3124 & 0.0797 & 0.0059 & 0.0002 & 0.0000 \\ 0.0000 & 0.0000 & 0.5590 & 0.3801 & 0.0563 & 0.0039 & 0.0006 \\ 0.0000 & 0.0000 & 0.0000 & 0.7096 & 0.2501 & 0.0304 & 0.0099 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.6964 & 0.1730 & 0.1306 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.2171 & 0.7829 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{pmatrix} \end{array}$$

$$p_{ij}^{6} = \begin{array}{cc} & \begin{array}{ccccccc} GA & AS & LI & LII & SL & RE & PRF \end{array} \\ \begin{array}{c} GA \\ AS \\ LI \\ LII \\ SL \\ RE \\ PRF \end{array} & \begin{pmatrix} 0.4387 & 0.4241 & 0.1158 & 0.0198 & 0.0011 & 0.0000 & 0,0000 \\ 0.0000 & 0.5436 & 0.3363 & 0.1087 & 0.0108 & 0.0006 & 0.0001 \\ 0.0000 & 0.0000 & 0.4977 & 0.4163 & 0.0776 & 0.0068 & 0.0016 \\ 0.0000 & 0.0000 & 0.0000 & 0.6625 & 0.2797 & 0.0399 & 0.0179 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.6478 & 0.1761 & 0.1761 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.1600 & 0.8400 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{pmatrix} \end{array}$$

$$p_{ij}^{7} = \begin{array}{cc} & \begin{array}{ccccccc} GA & AS & LI & LII & SL & RE & PRF \end{array} \\ \begin{array}{c} GA \\ AS \\ LI \\ LII \\ SL \\ RE \\ PRF \end{array} & \begin{pmatrix} 0.3824 & 0.4394 & 0.1440 & 0.0312 & 0.0023 & 0.0001 & 0,0000 \\ 0.0000 & 0.4911 & 0.3519 & 0.1385 & 0.0172 & 0.0012 & 0.0002 \\ 0.0000 & 0.0000 & 0.4430 & 0.4434 & 0.0998 & 0.0104 & 0.0284 \\ 0.0000 & 0.0000 & 0.0000 & 0.6187 & 0.3041 & 0.0489 & 0.0284 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.6026 & 0.1749 & 0.2225 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.1179 & 0.8821 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{pmatrix} \end{array}$$

$$p_{ij}^{8} = \begin{array}{cc} & \begin{array}{ccccccc} GA & AS & LI & LII & SL & RE & PRF \end{array} \\ \begin{array}{c} GA \\ AS \\ LI \\ LII \\ SL \\ RE \\ PRF \end{array} & \begin{pmatrix} 0.3340 & 0.4460 & 0.1707 & 0.0450 & 0.0043 & 0.0062 & 0,0000 \\ 0.0000 & 0.4437 & 0.3607 & 0.1679 & 0.0252 & 0.0021 & 0.0005 \\ 0.0000 & 0.0000 & 0.3944 & 0.4626 & 0.1222 & 0.0146 & 0.0062 \\ 0.0000 & 0.0000 & 0.0000 & 0.5776 & 0.3239 & 0.0572 & 0.2685 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.5605 & 0.1710 & 0.2685 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0869 & 0.9131 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{pmatrix} \end{array}$$

$$p_{ij}^{9} = \begin{array}{cc} & \begin{array}{ccccccc} GA & AS & LI & LII & SL & RE & PRF \end{array} \\ \begin{array}{c} GA \\ AS \\ LI \\ LII \\ SL \\ RE \\ PRF \end{array} & \begin{pmatrix} 0.2906 & 0.4456 & 0.1950 & 0.0607 & 0.0069 & 0.0005 & 0,0001 \\ 0.0000 & 0.4008 & 0.3639 & 0.1964 & 0.0346 & 0.0033 & 0.0011 \\ 0.0000 & 0.0000 & 0.3511 & 0.4752 & 0.0144 & 0.0193 & 0.0100 \\ 0.0000 & 0.0000 & 0.0000 & 0.5393 & 0.3396 & 0.0648 & 0.0563 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.5214 & 0.1651 & 0.3135 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0.640 & 0.9360 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{pmatrix} \end{array}$$

$$p_{ij}^{10} = \begin{array}{cc} & \begin{array}{ccccccc} GA & AS & LI & LII & SL & RE & PRF \end{array} \\ \begin{array}{c} GA \\ AS \\ LI \\ LII \\ SL \\ RE \\ PRF \end{array} & \begin{pmatrix} 0.2533 & 0.4398 & 0.2166 & 0.0781 & 0.0105 & 0.0008 & 0,0002 \\ 0.0000 & 0.3621 & 0.3627 & 0.2232 & 0.0452 & 0.0048 & 0.0019 \\ 0.0000 & 0.0000 & 0.3125 & 0.4823 & 0.1658 & 0.0243 & 0.0159 \\ 0.0000 & 0.0000 & 0.0000 & 0.5036 & 0.3516 & 0.0714 & 0.0734 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.4850 & 0.1580 & 0.3569 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0472 & 0.9528 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{pmatrix} \end{array}$$

.

$$p_{ij}^{218} = \begin{array}{cc} & \begin{array}{ccccccc} GA & AS & LI & LII & SL & RE & PRF \end{array} \\ \begin{array}{c} GA \\ AS \\ LI \\ LII \\ SL \\ RE \\ PRF \end{array} & \begin{pmatrix} 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.9991 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{pmatrix} \end{array}$$

$$p_{ij}^{219} = \begin{array}{cc} & \begin{array}{ccccccc} GA & AS & LI & LII & SL & RE & PRF \end{array} \\ \begin{array}{c} GA \\ AS \\ LI \\ LII \\ SL \\ RE \\ PRF \end{array} & \begin{pmatrix} 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.9992 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{pmatrix} \end{array}$$

$$p_{ij}^{220} = \begin{array}{cc} & \begin{array}{ccccccc} GA & AS & LI & LII & SL & RE & PRF \end{array} \\ \begin{array}{c} GA \\ AS \\ LI \\ LII \\ SL \\ RE \\ PRF \end{array} & \begin{pmatrix} 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0,9992 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{pmatrix} \end{array}$$

Hence the transition probability is stationary

3.2 Finding the expected number Academic Staff manpower Structure for 2030/2031 Academic session

(i) For 2021/2022 session

$$Q_1 = (19\ 52\ 49\ 35\ 26\ 12\ 15)\ {*}p_{ij}^{2}$$

$$p_{ij}^{2} = \begin{array}{cc} & \begin{array}{ccccccc} GA & AS & LI & LII & SL & RE & PRF \end{array} \\ \begin{array}{c} GA \\ AS \\ LI \\ LII \\ SL \\ RE \\ PRF \end{array} & \begin{pmatrix} 0.7599 & 0.2276 & 0.0124 & 0.0000 & 0.0000 & 0.0000 & 0,0000 \\ 0.0000 & 0.8161 & 0.1733 & 0.0106 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.7925 & 0.2003 & 0.0073 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.8718 & 0.1236 & 0.0046 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.8653 & 0.1164 & 0.0184 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.5429 & 0.4571 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{pmatrix} \end{array}$$

Ans. of Q_1= (14 46 48 40 27 9 20).

(ii) For 2022/2023 Academic session

$$Q_2 = (19\ 52\ 49\ 35\ 26\ 12\ 15)\ {*}p_{ij}^{3}$$

$$p_{ij}^{3} = \begin{array}{cc} & \begin{array}{ccccccc} GA & AS & LI & LII & SL & RE & PRF \end{array} \\ \begin{array}{c} GA \\ AS \\ LI \\ LII \\ SL \\ RE \\ PRF \end{array} & \begin{pmatrix} 0.6624 & 0.3030 & 0.0330 & 0.0014 & 0.0000 & 0.0000 & 0,0000 \\ 0.0000 & 0.7373 & 0.2331 & 0.0289 & 0.0007 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.7054 & 0.2740 & 0.0200 & 0.0005 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.8140 & 0.1728 & 0.0120 & 0.0012 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.8049 & 0.1461 & 0.0490 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.4000 & 0.6000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{pmatrix} \end{array}$$

Ans.: Q_2 = (12 44 47 43 27 9 23).

(i) For 2023/2024 Academic session.

$$Q_3 = (19\ 52\ 49\ 35\ 26\ 12\ 15)\ {*}p_{ij}^{4}$$

$$p_{ij}^{4} = \begin{array}{cc} & \begin{array}{ccccccc} GA & AS & LI & LII & SL & RE & PRF \end{array} \\ \begin{array}{c} GA \\ AS \\ LI \\ LII \\ SL \\ RE \\ PRF \end{array} & \begin{pmatrix} 0.5774 & 0.3586 & 0.0587 & 0.0049 & 0.0001 & 0.0000 & 0,0000 \\ 0.0000 & 0.6661 & 0.2781 & 0.0526 & 0.0026 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.6280 & 0.3333 & 0.0368 & 0.0018 & 0.0001 \\ 0.0000 & 0.0000 & 0.0000 & 0.7600 & 0.2174 & 0.0209 & 0.0044 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.7487 & 0.1638 & 0.7053 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.2947 & 0.7053 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{pmatrix} \end{array}$$

Ans.: Q_3 = (10 41 46 45 28 8 25).

(iv) For 2024/2025 Academic session

$$Q_4 = (19\ 52\ 49\ 35\ 26\ 12\ 15)\ {*}p_{ij}^{5}$$

$$p_{ij}^{5} = \begin{array}{c} \\ GA \\ AS \\ LI \\ LII \\ SL \\ RE \\ PRF \end{array} \begin{pmatrix} GA & AS & LI & LII & SL & RE & PRF \\ 0.5033 & 0.3980 & 0.0869 & 0.0110 & 0.0004 & 0.0000 & 0,0000 \\ 0.0000 & 0.6017 & 0.3124 & 0.0797 & 0.0059 & 0.0002 & 0.0000 \\ 0.0000 & 0.0000 & 0.5590 & 0.3801 & 0.0563 & 0.0039 & 0.0006 \\ 0.0000 & 0.0000 & 0.0000 & 0.7096 & 0.2501 & 0.0304 & 0.0099 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.6964 & 0.1730 & 0.1306 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.2171 & 0.7829 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{pmatrix}$$

Ans.: Q_4= (9 38 45 47 29 8 28).

(v) For 2025/2026 Academic session

$$Q_5 = (19\ 52\ 49\ 35\ 26\ 12\ 15) * p_{ij}^{6}$$

$$p_{ij}^{6} = \begin{array}{c} \\ GA \\ AS \\ LI \\ LII \\ SL \\ RE \\ PRF \end{array} \begin{pmatrix} GA & AS & LI & LII & SL & RE & PRF \\ 0.4387 & 0.4241 & 0.1158 & 0.0198 & 0.0011 & 0.0000 & 0,0000 \\ 0.0000 & 0.5436 & 0.3363 & 0.1087 & 0.0108 & 0.0006 & 0.0001 \\ 0.0000 & 0.0000 & 0.4977 & 0.4163 & 0.0776 & 0.0068 & 0.0016 \\ 0.0000 & 0.0000 & 0.0000 & 0.6625 & 0.2797 & 0.0399 & 0.0179 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.6478 & 0.1761 & 0.1761 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.1600 & 0.8400 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{pmatrix}$$

Ans.: Q_5 = (8 36 44 49 31 8 30).

(vi) For 2026/2027 Academic session

$$Q_6 = (19\ 52\ 49\ 35\ 26\ 12\ 15) * p_{ij}^{7}$$

$$p_{ij}^{7} = \begin{array}{c} \\ GA \\ AS \\ LI \\ LII \\ SL \\ RE \\ PRF \end{array} \begin{pmatrix} GA & AS & LI & LII & SL & RE & PRF \\ 0.3824 & 0.4394 & 0.1440 & 0.0312 & 0.0023 & 0.0001 & 0,0000 \\ 0.0000 & 0.4911 & 0.3519 & 0.1385 & 0.0172 & 0.0012 & 0.0002 \\ 0.0000 & 0.0000 & 0.4430 & 0.4434 & 0.0998 & 0.0104 & 0.0284 \\ 0.0000 & 0.0000 & 0.0000 & 0.6187 & 0.3041 & 0.0489 & 0.0284 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.6026 & 0.1749 & 0.2225 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.1179 & 0.8821 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{pmatrix}$$

Ans.: Q_6 = (7 33 42 51 32 8 32).

(vii) For 2027/2028 Academic session

$$Q_7 = (19\ 52\ 49\ 35\ 26\ 12\ 15) * p_{ij}^{8}$$

$$p_{ij}^{8} = \begin{array}{c} \\ GA \\ AS \\ LI \\ LII \\ SL \\ RE \\ PRF \end{array} \begin{pmatrix} GA & AS & LI & LII & SL & RE & PRF \\ 0.3340 & 0.4460 & 0.1707 & 0.0450 & 0.0043 & 0.0062 & 0,0000 \\ 0.0000 & 0.4437 & 0.3607 & 0.1679 & 0.0252 & 0.0021 & 0.0005 \\ 0.0000 & 0.0000 & 0.3944 & 0.4626 & 0.1222 & 0.0146 & 0.0062 \\ 0.0000 & 0.0000 & 0.0000 & 0.5776 & 0.3239 & 0.0572 & 0.2685 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.5605 & 0.1710 & 0.2685 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0869 & 0.9131 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{pmatrix}$$

Ans.: Q_7 = (6 31 41 52 33 8 34).

(viii) For 2028/2029 Academic session

$$Q_8 = (19\ 52\ 49\ 35\ 26\ 12\ 15) * p_{ij}^{9}$$

$$\boldsymbol{p_{ij}^{9}} = \begin{array}{c} \\ GA \\ AS \\ LI \\ LII \\ SL \\ RE \\ PRF \end{array} \begin{pmatrix} GA & AS & LI & LII & SL & RE & PRF \\ 0.2906 & 0.4456 & 0.1950 & 0.0607 & 0.0069 & 0.0005 & 0,0001 \\ 0.0000 & 0.4008 & 0.3639 & 0.1964 & 0.0346 & 0.0033 & 0.0011 \\ 0.0000 & 0.0000 & 0.3511 & 0.4752 & 0.0144 & 0.0193 & 0.0100 \\ 0.0000 & 0.0000 & 0.0000 & 0.5393 & 0.3396 & 0.0648 & 0.0563 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.5214 & 0.1651 & 0.3135 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0.640 & 0.9360 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{pmatrix}$$

Ans.: Q_7 = (5 29 39 53 34 8 36).

(ix) For 2029/2030 Academic session

$$Q_9 = (19\ 52\ 49\ 35\ 26\ 12\ 15)\ {*}p_{ij}^{10}$$

$$\boldsymbol{p_{ij}^{10}} = \begin{pmatrix} & GA & AS & LI & LII & SL & RE & PRF \\ GA & 0.2533 & 0.4398 & 0.2166 & 0.0781 & 0.0105 & 0.0008 & 0{,}0002 \\ AS & 0.0000 & 0.3621 & 0.3627 & 0.2232 & 0.0452 & 0.0048 & 0.0019 \\ LI & 0.0000 & 0.0000 & 0.3125 & 0.4823 & 0.1658 & 0.0243 & 0.0159 \\ LII & 0.0000 & 0.0000 & 0.0000 & 0.5036 & 0.3516 & 0.0714 & 0.0734 \\ SL & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.4850 & 0.1580 & 0.3569 \\ RE & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0472 & 0.9528 \\ PRF & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{pmatrix}$$

Ans.: $\boldsymbol{Q_9}$ = (4 27 38 54 35 8 39).

(x) For 2030/2031 Academic session

$$Q_{10} = (19\ 52\ 49\ 35\ 26\ 12\ 15)\ {*}p_{ij}^{11}$$

$$p_{ij}^{11} = \begin{pmatrix} & GA & AS & LI & LII & SL & RE & PRF \\ GA & 0.2387 & 0.3241 & 0.2058 & 0.0191 & 0.0020 & 0.0000 & 0{,}0000 \\ AS & 0.0000 & 0.5436 & 0.3323 & 0.1487 & 0.0008 & 0.0006 & 0.0020 \\ LI & 0.0000 & 0.0000 & 0.4977 & 0.4163 & 0.0776 & 0.0068 & 0.0016 \\ LII & 0.0000 & 0.0000 & 0.0000 & 0.6635 & 0.2297 & 0.0320 & 0.0141 \\ SL & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.6490 & 0.1721 & 0.1742 \\ RE & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.1600 & 0.8400 \\ PRF & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{pmatrix}$$

Ans.: $\boldsymbol{Q_{10}}$ = (3 25 37 55 36 7 42).

Table 9: Observed Values of Manpower Structure of the Academic Staff for Five (5) Academic Sessions

Year	GA	AL	L 2	L 1	S L	READER	PROF.
2016/2017	8	29	34	40	22	4	10
2017/2018	12	37	35	48	19	3	9
2018/2019	26	46	41	31	15	4	8
2019/2020	22	47	44	36	24	3	13
2020/2021	19	52	49	35	26	12	15

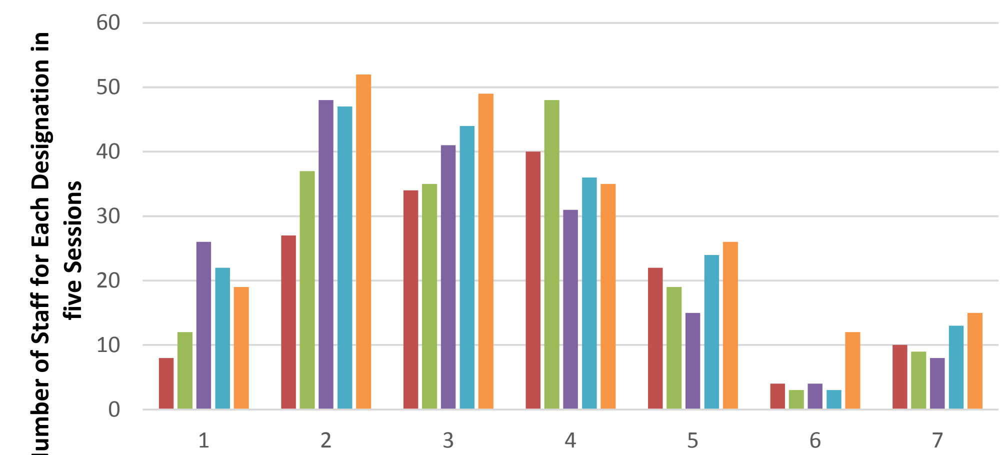

Fig 1: Observed Values of Manpower Structure of the Academic Staff for five (5) Academic Sessions…

Table 10: Predicted (Expected) Values of Manpower Structure of the Academic Staff for Ten (10) Academic Sessions

SESSIONS	GA	AL	LII	LI	SL	READER	PROF
2021/2022	14	46	48	40	27	9	20
2022/2023	12	44	47	43	27	9	23
2023/2024	10	41	46	45	28	8	25
2024/2025	9	38	45	47	29	8	28
2025/2026	8	36	44	49	31	8	30
2026/2027	7	33	42	51	32	8	42
2027/2028	6	31	41	52	33	8	34
2028/2029	5	29	39	53	34	8	36
2029/2030	4	27	38	54	35	8	39
2030/2031	3	25	37	55	36	7	42

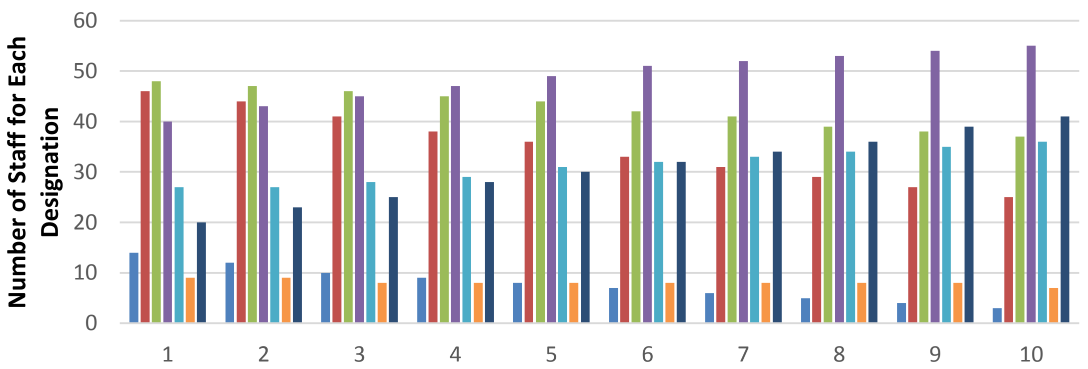

Fig 2: Predicted (Expectecd) Values of Manpower Structure of the Academic Staff for Ten (10) Academic Sessions

Summary and Conclusion

In this work, we obtained the transition probabilities matrix of Academic Staffs moving from grade i to grade j in each t^{th} session. The result of stationary test has shown that, the grade transition flows from one grade to the other is stationary, since it is stable at a particular state. Also, Table 9 has shown that at the beginning of 2016/2017 academic session $(t = 1)$, the observed staff structure of a Private University consisted 8 Graduate Assistants, 29 Assistant Lecturers, 34 Lecturer II, 40 Lecturer I, 22 Senior Lecturers, 4 Readers and 10 Professors if the current recruitment and promotion policies in the university remain unchanged. From the tabular and graphical representation of the predicted or expected structure of academic staff of the Private University for 2030/2031 Academic Session, as we have in Table 10 and Figure 2, it was observed that the distribution of the structure were given as 3 Graduate Assistants, 25 Assistant Lecturers, 35 Lecturer II, 55 Lecturer I, 36 Senior Lecturers, 7 Readers and 42 Professors if the current recruitment and promotion policies in the university remain unchanged. It was observed that, the number of Graduate Assistance, Assistance Lecturer, Lecturer II and Reader were decreased over the year. While the number of Lecturer I, Senior Lecturers and Professors were increased. In conclusion, it was observed that the institution was in need of more Lecturer I, Senior Lecturer and Professors than the number recruited. While it was also observed that the number of Graduate Assistance, Assistance Lecturer, Lecturer II and Readers were to be reduced to make the system effective.

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